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An undergraduate digital control project: a digital signal processor (DSP)-based magnetic levitation system

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ABSTRACT: In this article, the author proposes a digital control project of a digital signal processor (DSP)-based magnetic levitation system (MLS) for undergraduate education. The eZdsp F2812 was utilised as a digital controller, the pulse width modulation (PWM) technique was adopted to trigger the current driver and an infrared opto-electronic device was employed to sense the levitated object. The discrete-time system analysis was derived and the stability guaranteed by the root locus technology. These materials are considered to be appropriate for undergraduate control education.

INTRODUCTION

The magnetic levitation system (MLS) has been a famous undergraduate education tool from 1986 [1-7]. The MLS is also available from many equipment providers [8][9]. In this article, the author proposes an undergraduate control project of an MLS based on [2][3]. The main features of this MLS are digital signal processor (DSP)-based and pulse width modulation (PWM)-based.

With microprocessors becoming so fast, light, accurate and economical, control laws are often implemented in digital form. A special processor designed for real-time signal processing known as a DSP is particularly well-suited for use as an embedded digital controller [10-12]. The PWM is an important technique in which electrical consumption is low and efficiency is high. The eZdsp F2812 [12] is appropriate to control the MLS because the TMS320F2812 chip includes A/D interfaces with 12-bit resolution, a central processor with DSP kernel and PWM interfaces. In addition, the pack of the eZdsp F2812 provides Code Composer Studio (CCS) integrated developed environment (IDE) includes a text-editor, a C-compiler, an assembler, a debugger and a download tool. Control theories can be realised easily by using C-algorithms.

The other feature of this article is the *I-PD* controller. The *PD* function can stabilise the MLS [2][3]. The *I* function can eliminate or reduce the steady state error of the MLS. The mathematical derivation of an *I-PD* controller is proposed in detail in this article. These materials are appropriate for undergraduate control education.

MAGNETIC LEVITATION SYSTEM

The levitator is shown in Figure 1. The suspended object is a plastic ball and a permanent magnet has been bonded in it. The frame is made of wood so there is no magnetic field interference. The electromagnet is made of low carbon steel and the coil has been generated by utilising a 1.5 mm diameter copper wire coated with polyvinyl chloride.

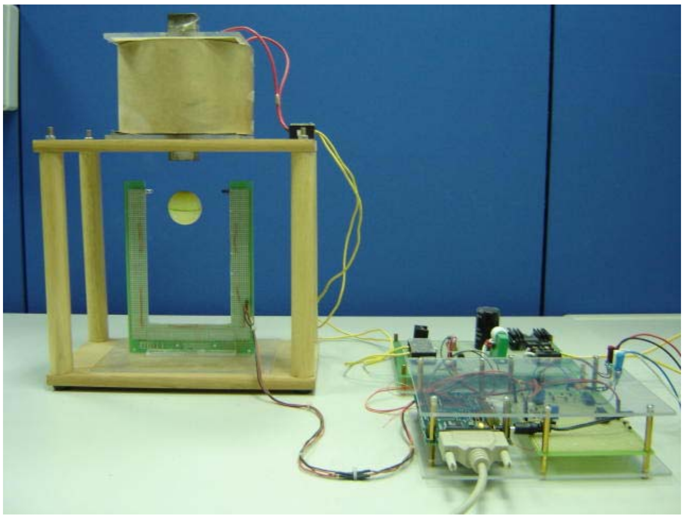

Figure 1: The experimental set-up of the MLS.

CURRENT DRIVER

The dc power supply is 18 volts and the coil resistor is 6 ohms, so the coil current is limited to 3 amps. The PWM frequency is 18.7 kHz and the average voltage is determined by the duty cycle. Because electromagnet coil is low-pass, it can attenuate the high frequency harmonics. Hence, the coil current is smooth. The linear factor of the current sensor circuit is set to be 1 volt/amp. Based on the design from ref. [3], the steady-state error of the coil current is zero and the settling time is 50 ms. Because the current dynamics is faster than the dynamics of the ball position, the current driver can be approximated simply by the unity gain.

POSITION SENSOR

Referring to Figure 1, a paired infrared emitter and receiver is used to sense the variation of the ball position. The output range is from 0.15 to 2.85 volts and depends upon the amount of lower shadow cast on a white diode. When the top of the ball is exactly at the centre of diodes, the measured voltage is 1.5 volts. The distance between the electromagnet and the top of the ball is 3.3 cm, with the coil current required to equalise the gravitational force being 1.48 amps.

DYNAMICS ANALYSIS

The system's dynamic equation can be obtained from [1-7] and rewritten as follows:

$$m\frac{d^2}{dt^2}x = mg - C\frac{i^2}{x^2} \tag{1}$$

where x is the distance between the electromagnet and the suspended ball, m is the mass of the suspended ball, g is the gravitational acceleration, i is the coil current and C is the force constant. Using the Taylor series expansion and neglecting all higher-order terms, the piece-wise linearised equation is:

$$m\frac{d^2}{dt^2}\delta x = -2C\frac{i_0}{x_0^2}\delta i + 2C\frac{i_0^2}{x_0^3}\delta x \tag{2}$$

where $\delta i = i - i_0$, $\delta x = x - x_0$ and the operating point (x_0, i_0) satisfies $mg = C i_0^2 / x_0^2$. By using of Laplace transform, equation (2) becomes:

$$G(s) = \frac{\delta X(s)}{\delta I(s)} = \frac{-2Ci_0 / mx_0^2}{s^2 - 2Ci_0^2 / mx_0^3} \tag{3}$$

where $\delta I(s)$ is the Laplace transform of δi and $\delta X(s)$ is the Laplace transform of δx.

The discrete-time system model of $G(s)$ can be obtained by the residue method [13]:

$$G(z) = \sum_{\substack{\text{at poles}\\ \text{of } G(\alpha)}} \left[\text{residues of } G(\alpha)\frac{1}{1-z^{-1}e^{T\alpha}}\right] \tag{4}$$

and can be presented as follows:

$$G(z) = \frac{\delta X(z)}{\delta I(z)} = \frac{-z\gamma\left(\beta^2-1\right)/\beta}{(z-\beta)(z-1/\beta)} \tag{5}$$

where $\beta = e^{T_0\sqrt{\frac{2Ci_0^2}{mx_0^3}}} > 1$ and $\gamma = \sqrt{\frac{C}{2mx_0}}$. The symbol T_0 is the sampling period. The linear factor of position sensor circuit is approximated by $-\rho\, volt/m$, so the plant can be described as follows:

$$\tilde{G}(z) = \frac{\delta\tilde{X}(z)}{\delta I(z)} = \frac{z\gamma\rho\left(\beta^2-1\right)/\beta}{(z-\beta)(z-1/\beta)} \tag{6}$$

where $\delta\tilde{X}(z)$ is the Z transform of the measured output $\delta\tilde{x}(k)$ of the position sensor.

TRACKING CONTROL

With reference to Figure 2, the transfer function of the inner loop subsystem is as follows:

$$T_{inner} = \frac{(z+\varphi)K_D\gamma\rho\left(\beta^2-1\right)/\beta}{(z-\beta)(z-1/\beta)+(z+\varphi)K_D\gamma\rho\left(\beta^2-1\right)/\beta} \tag{7}$$

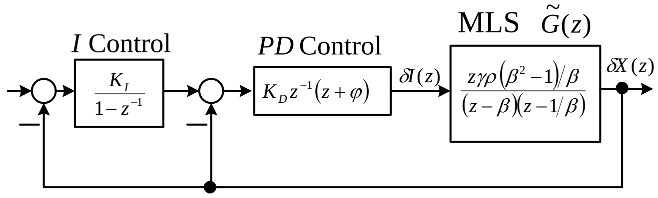

Figure 2: The function block diagram of the MLS.

The zeros of subsystem (7) are at $z_1 = -\varphi$ and $z_2 \to \infty$. The poles are the roots of the following characteristic equation:

$$\lambda_{inner} = 1 + \frac{(z+\varphi)K_D\gamma\rho\left(\beta^2-1\right)/\beta}{(z-\beta)(z-1/\beta)} \tag{8}$$

There are two stable root loci for the control parameters K_D and φ. In case 1, as shown in Figure 3, the zero $z=-\varphi$ lies between $z = 1/\beta$ and $z=\beta$. In case 2, as shown in Figure 4, the zero $z=-\varphi$ is at the left hand side of $z=1/\beta$. If one puts the poles of subsystem (7) at the real axis, then T_{inner} is becoming to:

$$T_{inner} = \frac{(z+\varphi)K_D\gamma\rho\left(\beta^2-1\right)/\beta}{(z-\sigma_1)(z-\sigma_2)} \tag{9}$$

where $\sigma_1 \in R$, $\sigma_2 \in R$ and $\sigma_1 < \sigma_2$. Now, we consider the characteristic equation of the overall system as follows:

$$1 + \frac{z(z+\varphi)K_I K_D\gamma\rho\left(\beta^2-1\right)/\beta}{(z-\sigma_1)(z-\sigma_2)(z-1)} = 0 \tag{10}$$

The open loop zeros are $z_1 = 0$, $z_2 = -\varphi$ and $z_3 \to \infty$. The open loop poles are $z_1 = \sigma_1$, $z_2 = \sigma_2$ and $z_3 = 1$. In case 1, as shown in Figure 3, if the left hand side pole of T_{inner} is negative ($\sigma_1 < 0$), then the root locus of (10) is plotted in Figure 5. In order to choose the integration parameter K_I appropriately, the overall system of Figure 2 is stable, ie the poles are all inside the unit circle. If the pole σ_1 is positive, then the root locus is shown in Figure 6. When the integration parameter K_I is selected appropriately, the stability of the overall system is guaranteed.

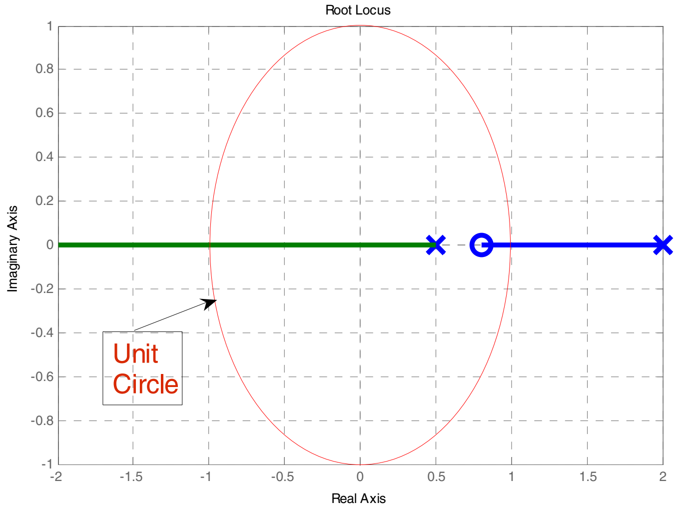

Figure 3: The root locus of case 1, where the zero $z=-\varphi$ is between $z=1/\beta$ and $z=\beta$.

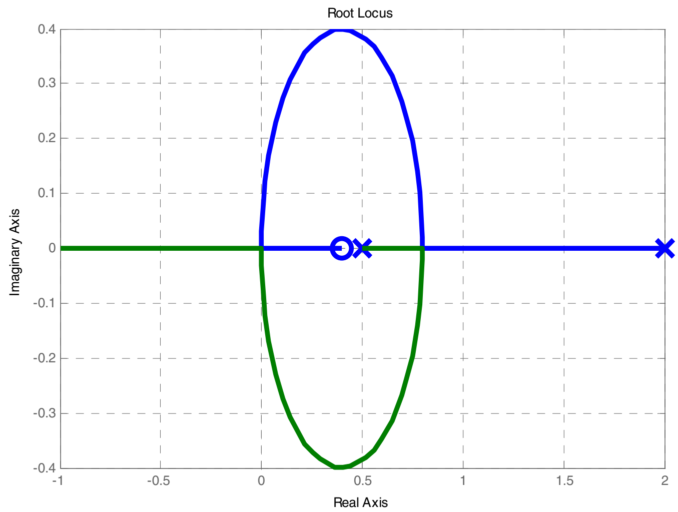

Figure 4: The root locus of case 2, where the zero $z=-\varphi$ is at the left hand side of $z=1/\beta$.

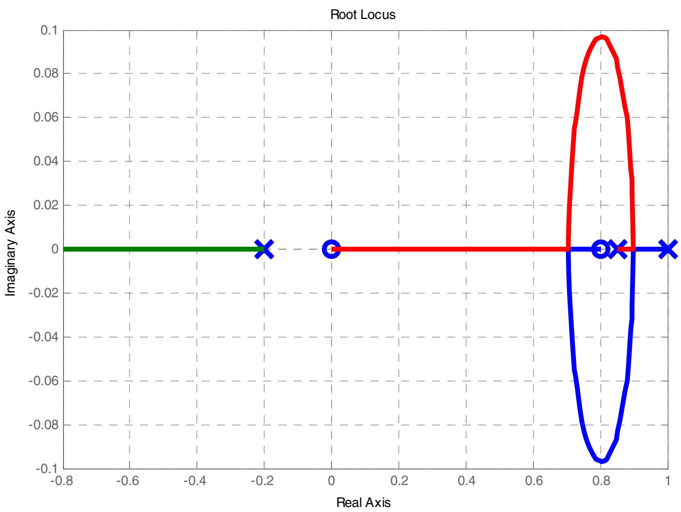

Figure 5: The root locus of (10), $\sigma_1<0$.

The experimental result is shown in Figure 7. There is no steady state error that has been detected with respect to the square wave input. The experimental data are listed in Table 1.

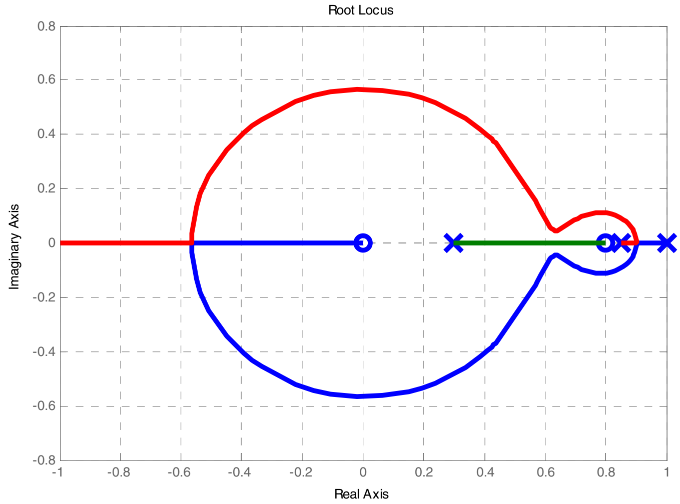

Figure 6: The root locus of (10), $\sigma_1>0$.

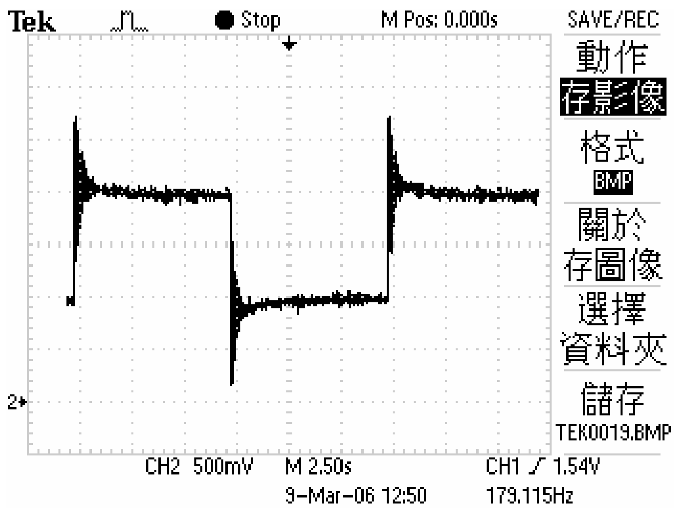

Figure 7: The square wave tracking response of the DSP-based MLS.

Table 1: Experimental data.

The integral gain K_I	0.5
The derivative gain K_D	2.0
The zero $z=-\varphi$	0.85
The bias current	1.48 A

CONCLUSION

In this article, the author describes the digital control project that has been developed for undergraduate education. The PWM-based current driver has been utilised in order to drive the electromagnet. A DSP-based platform has been used to control the system. The stabilising compensator was designed by the root locus methodology.

By the derivation shown in this article, the controller can be easily tuned; even the system parameters are not known a priori. These materials are considered to be appropriate for undergraduate control education.

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